Mathematical Sciences: MMSC4

LESSON 4:

UNIT STANDARD: 7448 & 7464

TOPIC: QUADRATIC PATTERNS

LESSON OBJECTIVES

1. Learners must be able to identify numeric (quadratic patterns correctly).
2. Learners should be able to recognise numeric (quadratic patterns correctly).
3. Learners should be able to classify patterns as decreasing or increasing.
4. Learners should be able extend or complete patterns.
5. Learners should be able to determine number of terms in the sequence.

Step: 1 Consider the following pattern:

1; 4; 9; 16; ………………………………………………………..

- Determine the next three terms?
  - ✓ I hope you notice that the next three terms are 1; 4; 9; 16; 25; 36; 49
- How did you get the next three terms?
  - ✓ I am sure, you may say that these are squares of natural numbers;

$T_1 = 1^2 = 1$

$T_2 = 2^2 = 4$

$T_3 = 3^2 = 9$

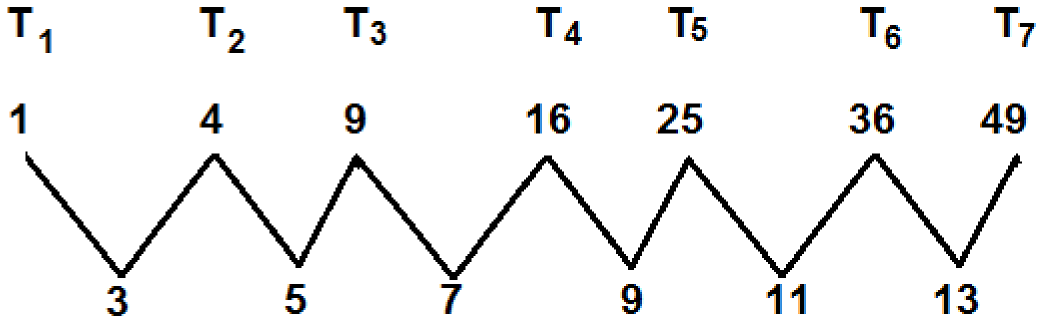
$T_4 = 4^2 = 16$

$T_5 = 5^2 = 25$

$T_6 = 6^2 = 36$

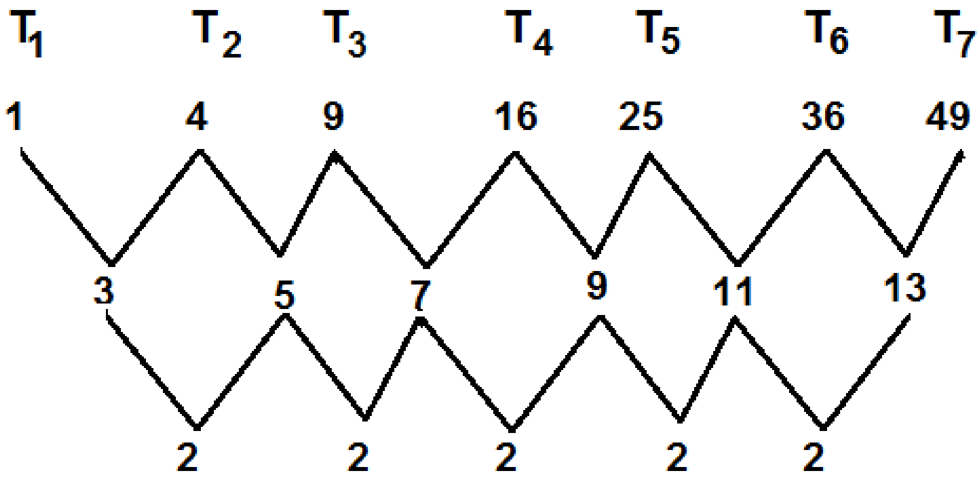
$T_7 = 7^2 = 49$

STEP: 2 However let us work out something interestingly different—their first differences;



- ✓ Therefore, their first differences are 3; 5; 7; 9; 11 and 13

STEP: 3 Now repeat the process by working out their second differences;

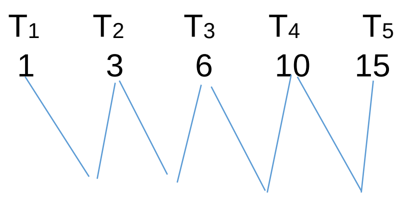


- ✓ You notice that their second difference is a constant 2

STEP: 4 Now let us consider the following pattern;

1; 3; 6; 10; 15 ………………………………………………….

- I hope you can see that these are not squares of natural numbers like the one above, I urge you to work out the first and second differences



2 3 4 5 → first difference

1 1 1 → second differences

STEP: 5 What kind of patterns are these?

- These are patterns whose second differences are always constant.
- Others define them as second difference patterns, just trying to hint on the very same point above
- Second difference patterns are called **QUADRATIC PATTERNS.**
- Therefore, a linear pattern has a constant first difference but a quadratic pattern has a constant second difference.
- Hence, the presence of common second difference in a pattern confirms that it is quadratic
- The general formula for any quadratic pattern is $T_n = an^2 + bn + \mathbf{c}$

STEP: 6 If we consider the quadratic formula $T_n = an^2 + bn + c$ and work out the first five terms thus by just substituting the n for the respective five terms, we get the following:

$T_1 = a(1)^2 + b(1) + c = a + b + c$

$T_2 = a(2)^2 + b(2) + c = 4a + 2b + c$

$T_3 = a(3)^2 + b(3) + c = 9a + 3b + c$

$T_4 = a(4)^2 + b(4) + c = 16a + 4b + c$

$T_5 = a(5)^2 + b(5) + c = 25a + 5b + c$

- Now our terms are as follows;

T_1	T_2	T_3	T_4	T_5
$a + b + c$	$4a + 2b + c$	$9a + 3b + c$	$16a + 4b + c$	$25a + 5b + c$

STEP: 7 Let us now work out our first differences:

$T_2 - T_1 = (4a + 2b + c) - (a + b + c)$

$= 4a + 2b + c - a - b - c$

$\underline{= 3a + b}$

$T_3 – T_2 = (9a + 3b + c) – (4a + 2b + c)$

$= 9a + 3b + c – 4a – 2b – c$

$= 5a + b$

$T_4 – T_3 = (16a + 4b + c) – (9a + 3b + c)$

$= 16a + 4b + c – 9a – 3b – c$

$= 7a + b$

$T_5 – T_4 = (25a + 5b + c) – (16a + 4b + c)$

$= 25a + 5b + c – 16a – 4b – c$

$= 9a + b$

STEP: 8 Now let us work out the second differences using their first differences.

i. $(5a + b) – (3a – b) = 5a + b – 3a – b = 2a$

ii. $(7a + b) – (5a + b) = 7a + b – 5a – b = 2a$

iii. $(9a + b) – (7a + b) = 9a + b – 7a – b = 2a$

- Now when we put our first and second differences below the terms, we have the following;

	T_1		T_2		T_3		T_4		T_5
	$a + b + c$		$4a + 2b + c$		$9a + 3b + c$		$16a + 4b + c$		$25a + 5b + c$
First difference		$3a + b$		$5a + b$		$7a + b$		$9a + b$	
Second difference			$2a$		$2a$		$2a$		

First difference

Second difference

- Now we can generalize out first difference as **3a + b** and the second difference as **2a**
- Therefore, in any quadratic pattern, take note of the following formulae

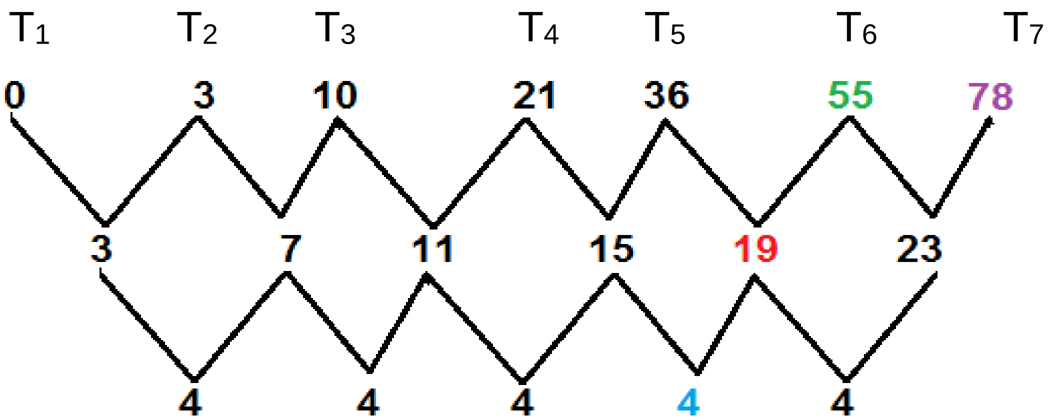
- ✓ **First Term** a + b + c
- ✓ **First difference** 3a + b
- ✓ **Second difference** 2a

STEP: 9 (Worked Example 1)

Consider the following pattern: 0; 3; 10; 21; 36

i. Determine the next two terms of the sequence?

- ✓ You can work out next two terms by using the first and second differences of the sequence. This is how you do it;



- ✓ We know that the second difference is always 4, hence we add this to the first difference of (T_5— T_4) thus 15 and lastly their sum (thus 19) to 36 which gives us 55 thus (36+ 19)
- ✓ In short this can be written as 4 + 15 + 36 = 55

Second difference **First difference**

- ➢ Therefore, the last term is 4 +19 +55 = 78

Worked example 2

- Now let us consider finding terms of a quadratic pattern when the general for-mula is given;
  - ✓ If the general formula for a pattern is $T_n = 3n^2 - 3n + 1$, determine the first three three terms of the sequence

- ✓ One can work out the first three terms by just substituting (1) thus T_1, (2) thus T_2

(3) thus T_3 in the formula as follows;

$T_n = 3n^2 - 3n + 1 = T_1 = 3 \times 1^2 - 3x1 + 1 = 3 - 3 + 1 = 1$

$T_n = 3n^2 - 3n + 1 = T_2 = 3 \times 2^2 - 3 \times 2 + 1 = 12 - 6 + 1 = 7$

$T_n = 3n^2 - 3n + 1 = T_3 = 3 \times 3^2 - 3 \times 3 + 1 = 27 - 9 + 1 = 19$

Worked example 3

- Now let us consider finding number of terms in a quadratic sequence.
- Consider the following quadratic sequence 1; 4; 9; 16 -----------81,
- If the general formula of the above sequence is $T_n = n^2$, how many terms are

In the sequence.

- Formulate an equation of the last term in the sequence and equate it to the **n^th term^** (general formula)

$T_n = n^2 = 81$

$n^2 = 81$

$n = \sqrt{81}$

$\underline{n = 9}$

- ✓ Is the pattern increasing or decreasing? Give a reason for your answer?
- ✓ It is increasing because the value of every next term is more than the previous.

CONCLUSION

- Quadratic patterns are sequences with a constant second difference.
- It is the constant second difference which confirms that the pattern is quadratic,
- We can determine terms in a quadratic pattern, given some consecutive terms by adding the sum of the first and second differences to the previous term of the sequence.
- When the n^{th} term is given like $T_n = 3n^2 - 3n$, terms in the sequence can be worked out by just substituting n for 1 thus T_1, , 2 thus T_2 etc in the formula.

- When you are given a sequence that includes the last term and its relative general formula(n^{th} term), you can find number of terms in the sequence by just formulating an equation that equates the last term to the general formula.

CONSOLIDATION (Problems for practice)

1. Consider the following pattern;
   -3; 1; 7; 15; 25.
   a) Determine the next two terms of the sequence
   b) Is the pattern increasing or decreasing, give a reason for your answer
2. The general formula for a pattern is $T_n = 6(n-1)$.
   Determine the first three terms of the pattern
3. Consider the following pattern ; 1; ¼; 1/9; 1/16--------------1/64.
   IF the n^{th} term of the above sequence is $1/n^2$, how many terms are in the sequence.
   Is the pattern increasing or decreasing? Motivate your answer?

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